15. Ideal Gases

15.3 Kinetic theory of gases

Paper 4

Question Paper

1 **(a)** **(ii)** Use one of the basic assumptions of the kinetic theory to explain what can be deduced about the potential energy associated with the random motion of molecules in an ideal gas.

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..... [2]

(b) A sample of 0.26 m^3 of an ideal gas is at pressure 2.0×10^5 Pa and temperature 290 K.

Determine:

(ii) the average translational kinetic energy E_K of one molecule of the gas

E_K = ... J [2]

2 **(b)** Two cylinders X and Y each contain a sample of an ideal gas. The samples are in thermal equilibrium with each other.

X has a volume of $0.0260\,m^3$ and contains 0.740 mol of gas at a pressure of 1.20×10^5 Pa. Y has a volume of $0.0430\,m^3$ and contains gas at a pressure of 2.90×10^5 Pa. Data for the two cylinders are shown in Fig. 2.1.



Fig. 2.1

(iii) The gas in X consists of molecules that each have a mass that is four times the mass of a molecule of the gas in Y.

Explain how the root-mean-square (r.m.s.) speed of the molecules in X compares with the r.m.s. speed of the molecules in Y.

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..... [3]

3 **(b)** Two samples X and Y of ideal gases are both at thermodynamic temperature T.

Sample X has volume V and consists of N molecules, each of mass m.
Sample Y has volume $2V$ and consists of $2N$ molecules, each of mass $2m$.

(ii) The temperature of sample X is now varied.

On Fig. 3.1, sketch the variation with thermodynamic temperature of the root-mean-square (r.m.s.) speed of the molecules of the gas.

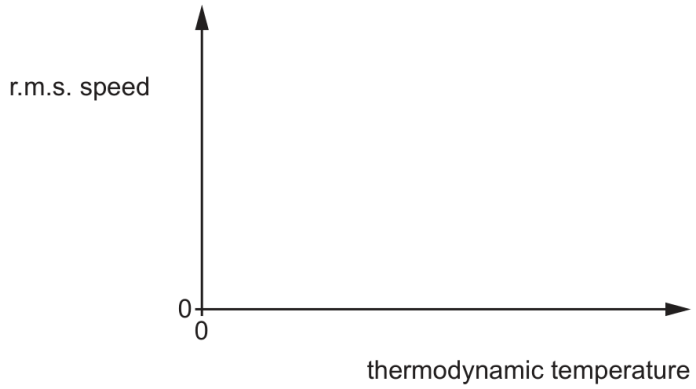


Fig. 3.1

[2]

4 **(a)** State **three** of the basic assumptions of the kinetic theory of gases.

1 .....

.....

2 .....

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3 .....

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[3]

(b) Explain how molecular movement causes the pressure exerted by a gas.

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..... [3]

5 **(a)** State **two** of the basic assumptions of the kinetic theory of gases.

1 .....

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2 .....

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[2]

6 **(b)** A sealed vessel contains a mass of 0.0424 kg of an ideal gas at 227 °C.
The pressure of the gas is 1.37×10^5 Pa and the volume of the gas is 0.640 m^3.

Calculate:

(iii) the root-mean-square (r.m.s.) speed v of the molecules of the gas.

v = ... $m\,s^{-1}$ [3]

(c) The gas in **(b)** is now cooled gradually to absolute zero.

On Fig. 2.1, sketch the variation with thermodynamic temperature T of the r.m.s. speed of the molecules of the gas.

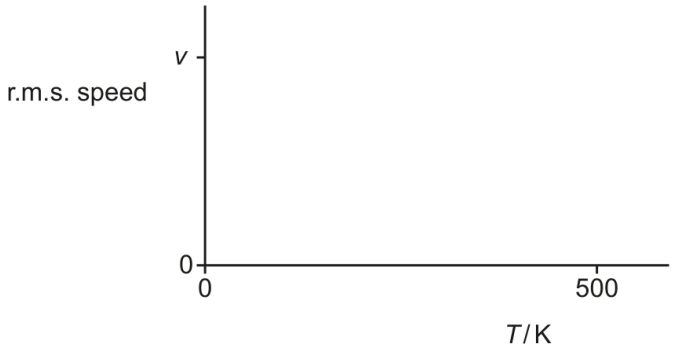


Fig. 2.1

[2]

7 **(a)** The product pV for an ideal gas is given by

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

where p is the pressure of the gas and V is the volume of the gas.

(i) State the meaning of the symbols N, m and $\langle c^2\rangle$ in this equation.

N: .....

m: .....

$\langle c^2\rangle$: .....

[3]

(ii) Use the equation of state for an ideal gas to show that the average translational kinetic energy E_K of a molecule of the gas at thermodynamic temperature T is given by

$$E_K = \frac{3}{2}kT.$$

[2]

(b) The surface of a star consists mainly of a gas that may be assumed to be ideal. The molecules of the gas have a root-mean-square (r.m.s.) speed of $9300\,\text{m}\,\text{s}^{-1}$.

The mass of a molecule of the gas is 3.34×10^{-27} kg.

Determine, to three significant figures, the temperature of the surface of the star.

temperature = ... K [2]

(d) The gas at the surface of a star has a very high pressure.

Use the basic assumptions of the kinetic theory to suggest why, in practice, a gas at the surface of a star is unlikely to behave as an ideal gas.

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..... [2]

8 **(a)** **(ii)** The Moon may be considered to be an isolated uniform sphere of mass 7.3×10^{22} kg and radius 1.7×10^{6} m.

Calculate the gravitational potential at the surface of the Moon. Give a unit with your answer.

(b) An isolated uniform spherical planet has gravitational potential ϕ at its surface.

A particle of mass m is projected vertically upwards from the surface. The particle is given just enough kinetic energy to travel to an infinite distance away from the planet, escaping from the gravitational pull of the planet, without any additional work being done on it.

(ii) Show that the speed v at which the particle is projected upwards from the surface of the planet is given by

$$v = \sqrt{-2\phi}.$$

(c) A particle is moving upwards at the surface of the Moon.

Use your answer in **(a)(ii)** and the expression in **(b)(ii)** to determine the minimum speed of this particle that will result in it escaping from the gravitational pull of the Moon.

(d) Hydrogen may be assumed to be an ideal gas.
The mass of a hydrogen molecule is 3.34×10^{-27} kg.

Calculate the root-mean-square (r.m.s.) speed of a hydrogen molecule in hydrogen gas that is at a temperature of 400 K.

r.m.s. speed = ... $m\,s^{-1}$ [3]

(e) The surface of the Moon reaches temperatures of approximately 400 K when in direct sunlight.

Use your answers in **(c)** and **(d)** to suggest a reason why the Moon does not have an atmosphere consisting of hydrogen.

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..... [1]

9 **(b)** Use the equation in **(a)** to show that the average translational kinetic energy E_K of a molecule of an ideal gas is given by

$$E_K = \frac{3}{2}kT.$$

[2]

(c) The mass of an oxygen molecule is 5.31×10^{-26} kg. Assume that oxygen behaves as an ideal gas.

(i) Use the equation in **(b)** to determine the root-mean-square (r.m.s.) speed u of an oxygen molecule at 23 °C.

u = ... $m\,s^{-1}$ [3]

(ii) A fixed mass of oxygen gas at initial pressure *P* is sealed in a cylindrical container by a movable piston at one end, as shown in Fig. 3.1.

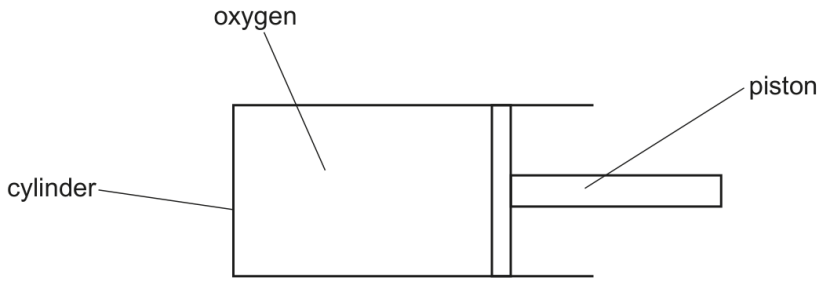


Fig. 3.1

The temperature of the gas is 23 °C.

The piston is slowly moved into the cylinder so that the oxygen gas is compressed. At all times, the gas and the container remain in thermal equilibrium with the surroundings.

On Fig. 3.2, sketch the variation with pressure of the r.m.s. speed of the oxygen molecules as the pressure increases.



Fig. 3.2

[2]

10 An ideal gas is contained in a cylinder by means of a movable frictionless piston, as illustrated in Fig. 2.1.

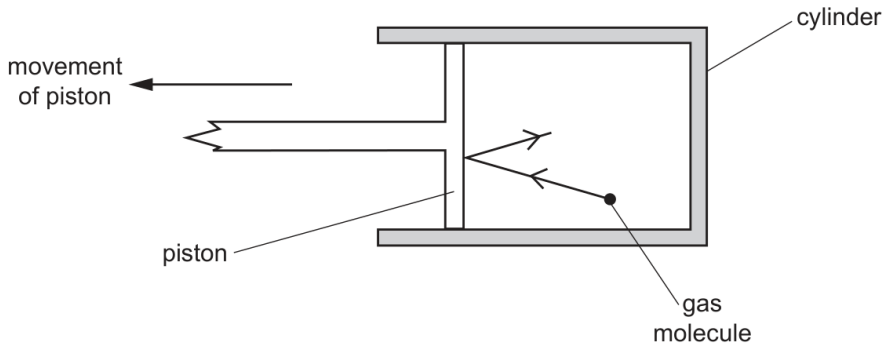


Fig. 2.1

Initially, the gas has a volume of $1.8 \times 10^{-3}\,m^3$ at a pressure of $3.3 \times 10^5\,Pa$ and a temperature of 310 K.

(b) Use kinetic theory to explain why, when the piston is moved so that the gas expands, this causes a decrease in the temperature of the gas.

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..... [3]

11 (b) A sealed container of fixed volume V contains N molecules, each of mass m, of an ideal gas at pressure p.

(ii) Show that the mean translational kinetic energy E_K of a molecule of the gas is given by

$$E_K = \frac{3}{2}kT.$$

[2]

12 **(a)** One of the assumptions of the kinetic theory of gases is that all collisions involving molecules of the gas are elastic.

(i) State what is meant by an *elastic* collision.

.....

..... [1]

(ii) State **two** other assumptions of the kinetic theory of gases.

1. .....

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2. .....

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[2]

(b) A molecule of an ideal gas has mass m and is contained in a cubic box of side length L. The molecule is moving with velocity u towards the face of the box that is shaded in Fig. 3.1.

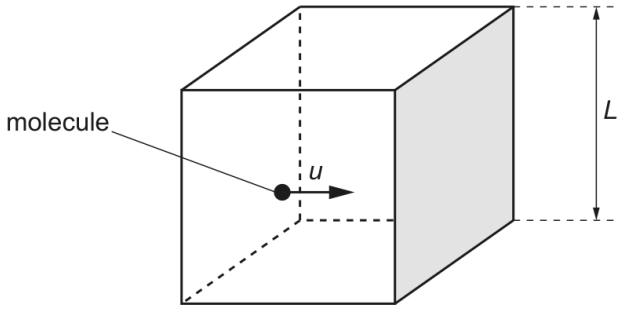


Fig. 3.1

The molecule collides elastically with the shaded face and the face opposite to it alternately.

Deduce expressions, in terms of m, u and L, for:

(i) the magnitude of the change in momentum of the molecule on colliding with a face

change in momentum = ... [1]

(ii) the time between consecutive collisions of the molecule with the shaded face

time = ... [1]

(iii) the average force exerted by the molecule on the shaded face

force = ... [1]

(iv) the pressure on the shaded face if the force in **(iii)** is exerted over the whole area of the face.

pressure = ... [1]

(c) When the model described in **(b)** is extended to three dimensions, and to a gas containing N molecules, each of mass m, travelling with mean-square speed $\langle c^2 \rangle$, it can be shown that

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

where p is the pressure exerted by the gas and V is the volume of the gas.

Use this expression, together with the equation of state of an ideal gas, to show that the average translational kinetic energy E_K of a molecule of an ideal gas is given by

$$E_K = \frac{3}{2}kT$$

where T is the thermodynamic temperature of the gas and k is the Boltzmann constant.

[2]

(d) The mass of a hydrogen molecule is 3.34×10^{-27} kg.

Use the expression for E_K in **(c)** to determine the root-mean-square (r.m.s.) speed of a molecule of hydrogen gas at 25 °C.

r.m.s. speed = ... $m\,s^{-1}$ [2]

13 (a) A square box of volume V contains N molecules of an ideal gas. Each molecule has mass m.

Using the kinetic theory of ideal gases, it can be shown that, if all the molecules are moving with speed v at right angles to one face of the box, the pressure p exerted on the face of the box is given by the expression

$$pV = Nmv^2. \qquad \text{(equation 1)}$$

This expression leads to the formula

$$p = \frac{1}{3}\rho\langle c^2\rangle \qquad \text{(equation 2)}$$

for the pressure p of an ideal gas, where ρ is the density of the gas and $\langle c^2\rangle$ is the mean-square speed of the molecules.

Explain how each of the following terms in equation 2 is derived from equation 1:

ρ: .....

.....

$\frac{1}{3}$: .....

.....

$\langle c^2\rangle$: .....

.....

[4]

14 **(a)** The kinetic theory of gases is based on a number of assumptions about the molecules of a gas.

State the assumption that is related to the volume of the molecules of the gas.

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.....

..... [2]

(b) An ideal gas occupies a volume of $2.40 \times 10^{-2}\,m^3$ at a pressure of $4.60 \times 10^5\,Pa$ and a temperature of 23 °C.

(ii) Each molecule has a diameter of approximately $3 \times 10^{-10}\,m$.

Estimate the total volume of the gas molecules.

volume = ... m^3 [3]

(c) By reference to your answer in **(b)(ii)**, suggest why the assumption in **(a)** is justified.

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..... [1]

15 (a) Smoke particles are suspended in still air. Brownian motion of the smoke particles is seen through a microscope.

Describe:

(i) what is seen through the microscope

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..... [1]

(ii) how Brownian motion provides evidence for the nature of the movement of gas molecules.

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..... [2]

16 **(a)** Use one of the assumptions of the kinetic theory of gases to explain why the potential energy of the molecules of an ideal gas is zero.

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.....[1]

(b) The average translational kinetic energy E_K of a molecule of an ideal gas is given by the expression

$$E_K = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

where m is the mass of a molecule and k is the Boltzmann constant.

State the meaning of the symbol

(i) $\langle c^2\rangle$,

.....[1]

(c) A cylinder of constant volume $4.7 \times 10^4\,\text{cm}^3$ contains an ideal gas at pressure $2.6 \times 10^5\,\text{Pa}$ and temperature 173 °C.

The gas is heated. The thermal energy transferred to the gas is 2900 J. The final temperature and pressure of the gas are T and p, as illustrated in Fig. 2.1.



Fig. 2.1

(i) Calculate

1. the number N of molecules in the cylinder,

N = ...[3]

2. the increase in average kinetic energy of a molecule during the heating process.

increase = ... J [1]

(ii) Use your answer in **(i) part 2** to determine the final temperature T, in kelvin, of the gas in the cylinder.

T = ... K [3]

17 (b) An ideal gas comprised of single atoms is contained in a cylinder and has a volume of $1.84 \times 10^{-2}\,m^3$ at a pressure of $2.12 \times 10^{7}\,Pa$.
The mass of gas in the cylinder is 3.20 kg.

(i) Determine, to three significant figures, the root-mean-square (r.m.s.) speed of the atoms of the gas.

r.m.s. speed = ...$m\,s^{-1}$ [3]